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STEM

SAT Mathematics Demystified

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Table of Contents

Goals and Objectives

The primary objective of this project is to demonstrate how SAT mathematics content and SAT test taking strategies can be seamlessly embedded into the current highschool math curriculum. The goal is to offer guidance on how to effectively create an environment in the classroom and beyound to encourage more critical thinking about mathematics.

According to The College Board (2022), the national average SAT Mathematics score for 2021 was 538 (out of 800 points). The 2021 SAT Mathematics scores for Black and Hispanic students were 477 and 490 respectively; this is an exponential drop.

This project aims to eliminate the SAT Math gap that exists between the various demographic groups and to simultaneously increase the SAT Mathematics profile of all students regardless of ethnic background or race.

Teachers will learn how to seamlessly incorporate and highlight SAT content that is already present in the curriculum that they are currently teaching. They will be taught how to do this without course disruption or student confusion.

Florida Standards

MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of exponents.

MA.912.AR.3.5
Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.

MA.912.AR.3.6
Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Course Outline/Overview

The mathematics that's tested on the SAT exam is mainly Algebra, Geometry, and Statistics. These topics are further subdivided into: The Heart of Algebra; Problem Solving and Data Analysis; Passport to Advanced Mathematics; and Additional Topics in Mathematics. The links below provide information from each sub section

This College Board link provides teachers and students with valuable testing and registration information.
https://collegereadiness.collegeboard.org/pdf/official-sat-study-guide-ch-22-additional-topics-math.pdf

SAT mathematics practice is effective when students are faithfully engaged in independent practice; extracurricular practice; and in teacher guided practice. Teachers will be taught how to maximize student outcomes in all three of these areas.

Independent Practice:
- Independent practice is the cornerstone to SAT success.
- Parental involvement is a key element to ensuring that essential independent practice is occuring.
- Schedule setting should be a collaborative effort between the student and the parent.
- SAT information should be readily shared with parent groups and organizations at school.
- Provide online resources for students, use social media for announcements, and post flyers.
- Help parents understand their student scores.

Extracurricular Practice:
- Students can practice in the following ways: Create an SAT club; After-school sessions; or use a community based organization.
- Incorporate SAT practice tools during non content classes like leadership, study hall, or college prep.
- Create a partnership with a community based college access organization to offer a place to practice.
- The Houston Independent School District ran a District Wide SAT initiative

Teacher Guided Practice:
- Sponsor an official SAT Club that meets consistently during the six weeks leading to the SAT exam.

- Always use Official College Board SAT materials.
- Practice on a daily basis.
- Assign SAT homework (at least 60 minutes per week).

Building an effective teacher guided SAT program requires knowledge of the content, stamina and grit, minimal resources, and a clear understanding of the impact of an improved SAT score on a student's life.

The acquisition of content knowledge comes from initially understanding the objectives (College Board) of the SAT mathematics test. Secondly, there is a requirement for educators to actually do all of the SAT math problems that they can possibly find. This activity of doing problems will allow educators to experience, see, and know multiple points of access for the limited problem types that students will encounter on the actual exam.

Implicit and Explicit SAT Mathematics Preparation During Classroom Instruction

There are endless opportunities in the Algebra I and the Algebra II curriculum to point out and clarify SAT content. Students must be well versed in all of the vocabulary that is associated with the different content areas. Additionally, they must be able to seamlessly construct equations from word problems, to quickly and effectively analyze graphs, and to navigate and synthesize the content of tables.

Ratios, proportions, and rates are fundamental components of the math SAT. It is imperative that students understand these ideas intuitively. Therefore, teachers must use every opportunity to ensure that students are given ample time to explore and clarify the similarities and differences between these three ideas.

When students are being introduced to the idea of linear functions in algebra I, this is an opportune time to emphasize important key ideas that are tested on the SAT; linear functions and expressions are everywhere on the exam. Students should be able to correlate a starting point in a word problem with the y-intercept in a linear function. Also, it should be emphasized that the slope (m) of a linear function is the rate at which the dependent variable is changing.

When systems of equations are introduced in the classroom (linear and quadratic or a combination of both), teachers must not only passionately teach the elimination and

substitution methods. They should further emphasize the graphical representation of what a solution to the system actually means (no solution, one solution, and infinitely many solutions). These ideas are implicitly and explicitly tested on the SAT. Additionally, the classroom can be effectively used to teach students how to identify and construct systems of equations from word problems and to subsequently solve them.

The teaching of functions and their graphs is perhaps the best time to demonstrate to students how a mathematical function and its graph are different versions of the same thing. The emphasis should be placed on the linear, quadratic, cubic, and exponential functions. Students should understand the concepts of zeros, multiplicities, x-intercepts, where a function is increasing and decreasing, where its negative and positive, and how these characteristics are manifested on a graph. The ability to read and understand what a graph is doing is a priceless skill for the SAT and beyond.

The Fundamental Theorem of Algebra, the Remainder Theorem, and the Factor Theorem should all be thoroughly investigated during classroom instruction in order to ensure deep student comprehension. Ideas related to these theorems are routinely present on the SAT exam. Students should have a formal understanding of what these theorems mean and how they can be expressed in the form of a question. Often, knowledge of these theorems reduces the time spent on an SAT question to seconds instead of minutes. Emphasis on the practice of long and synthetic division is also a useful idea.

A student's formal introduction to factoring is an opportune time for them to permanently master a key component of SAT mathematics. This idea should be built up from understanding the greatest common factor, to grouping, and using the quadratic formula. Students should also be explicitly taught the purpose of the discriminant in a functional way. Like the theorems mentioned above, knowledge of the discriminant can reduce the response time to certain questions on the actual SAT exams to seconds. Additionally, the practice of completing the square must be emphasized because it manifests itself in multiple ways on the exam.

When teaching geometry the following concepts should be highlighted: the properties of similar triangles; characteristics of parallel lines and transversals, the trigonometric characteristics of triangles (30, 60, 90 and 45, 45, 90); properties of circles and their equations; determining the number of degrees in a polygon.

When teaching statistics and probability in the classroom emphasis should be placed on the following: understanding measures of center (mean, median, and mode); dot plots,

histograms, line graphs, box and whisker plots; and frequency and range. Students also need to understand and be able to identify the concept of bias as it relates to surveys.

The content that is mentioned here is not an exhaustive list. However, emphasis and proper preparation on these topics can give each student a fighting chance to maximize their SAT mathematics potential.

Comparative Analysis of SAT and EOC Math Questions

Formative questions are rarely test items on the SAT, the questions are almost always summative in nature. The table below provides a comparative analysis of SAT Math questions and Algebra EOC questions.

	SAT Mathematics	Algebra EOC
#1	#1) If $\dfrac{x^{a^2}}{x^{b^2}} = x^{16}$, $x > 1 \, and \, a + b \, = \, 2$, $what \, is \, the \, value \, of \, a - b$	$Divide \, (3x^5 + 12x^4 - 21x^2)$ by $6x$
#2	$ax \, + \, by \, = \, 12$ $2x + 8y \, = \, 60$ In the system of equations above, a and b are constants. If the system has infinitely many solutions, what is the value of $\dfrac{a}{b}$	$Solve$: $m - 8 \, \geq \, 14$
#3	A radioactive substance decays at an annual rate of 13 percent. If the initial amount of the substance is 325 grams, which of the following functions f models the remaining amount of the substance, in grams, t years later?	$Solve \, y \, + \, w \, - \, \dfrac{3}{4}z = \, 0$ for z

In example #1 the following is required for the SAT Math question:
- Knowledge of the rules to divide monomials.
- Knowledge of rules to multiply monomials with like bases.
- Inherent understanding that the left side of an equation is equal to the right side of the equation.

- The ability to factor quadratic expressions.
- The ability to factor 16.
- The ability to know that the factors in the exponent on the left are equal to the factors in the exponent on the right.
- The knowledge to correctly use the information given (a+b =2)
- The ability to do this in about 1 minute and 16 seconds.

Example #1 Algebra EOC Question requirements:

- Know the rules related to the division of monomials

In example #2 the following is required for the SAT Math question.
- The student has to have an initial understanding of systems of equations.
- The student would need to know what a solution for a system represents.
- The student would have to know what a system that has infinitely many solutions means analytically.
- The student would have to know the parts of a linear function.
- The student would have to understand the concept of slope and y-intercepts
- The student would need to know how to simplify each equation to identify their slopes.
- This must be accomplished in about 1 minute and 16 seconds.

Example #2 Algebra EOC Question requirements:
- The student needs to know how to solve a one step inequality.

In example #3 the following is required for the SAT Math question.
- The student must know the structure and the components of the exponential growth function.
- The student must be able to differentiate between exponential growth and exponential decay.
- The student must know that a decay of 13% will likely show up as 0.87 in the exponential expression.
- The student must be aware that the initial amount precedes the decay factor and is not connected to the exponent.
- This must be done in one minute and 16 seconds.

Example #3 Algebra EOC Question requirements:
- The student needs to know how to solve a three step equation using the rules of equality.

It is imperative that teachers realize that the SAT does not resemble State summative exams. There is no meaningful correlation between success on a state's End of Course Exams and the SAT Exam. The rigor and the critical thinking, as evidenced in the above examples, that is required on the SAT is almost entirely absent on State exams. This is primarily because the SAT is the de facto "college entrance" exam while State exams serve more of an accountability role (to taxpayers).

Building The SAT Math Program

Perhaps the chief reason why students struggle on the mathematics portion of the SAT exam is because the math rigor is not present in most classrooms and curriculum to properly prepare them for the exam. A student can only improve their SAT math score by developing maturity in mathematics. The necessary math maturity can be successfully built through the following; self directed independent practice; extracurricular practice with a cohort of serious people; and teacher directed practice. The teacher guided practice should be at the forefront of such a system.

All of the instructional materials that a teacher uses must be consistent with The College Board's standards. There are many old SAT tests, practice tests and test questions that are available free of charge. These are typically accompanied by explicit answers to each question. This bank of questions provides more than enough materials to build a successful program.

The teacher led sessions should be no longer than one hour and 30 minutes. This provides enough time to administer a portion of a test, thoroughly review each question, and additionally give a short lecture on a specific topic (factoring, finding zeros, reading graphs, etc). The lecture should occur at the beginning of the session and should never be more than 15 minutes. The students should always take a practice test that should last for no more than 25 minutes. The remaining 50 minutes of the session should be used to thoroughly explain each question and to encourage student discussion and participation. The review should always begin with the questions that had the highest percentage of incorrect responses and progress to the questions that were more successfully answered. This requires a real time tracking program (software) that can gather data and simultaneously disaggregate the data.

Practice exams can be effectively stored in a program like Padlet (Padlet.com). Here students can be given access to the practice tests and questions at the appropriate time. They can even

access these practice tests using their phones. During the assessment portion of the teacher led sessions students must record their responses in a program like Google Forms. The initial investment for the teacher is to create an answer key for each practice exam. Google Forms will then collect the student responses in real time during the assessment and disaggregate the data into multiple categories. One such category is "The Frequently Missed Questions." The data from this category can then be used to guide the review and will give the teacher the ability to place emphasis on specific pertinent questions.

The rule of thumb is that all students in the program should have registered for an upcoming SAT exam. This builds positive pressure for the student and incentivizes their efforts. Additionally, seniors who are at the cusp of applying to college should not be in the same cohort with juniors. The teacher led sessions should begin exactly six weeks prior to the SAT exam. During this time there should be two cohort meetings per week where all of the students in the cohorts are expected to attend. This scenario would allow for 12 teacher led sessions. There is anecdotal evidence that 10 to 12 sessions seem to bring a certain degree of success.

Tangible Success

Students who have diligently engaged in this program have increased their SAT Math scores on average by over 100 points. There are a few students who have come very close to earning perfect scores on the math portion of the exam. Over 90 % of these students qualify for The Bright Futures Scholarship. Committed students have even manifested a tangible change in the way they approach other academic subjects once they have completed this program.

Students who have successfully completed this program are at major universities around the country. A few of the schools are listed here: Brown University; Tulane University; Georgia Tech University; Wheaton College; Emory University; The University of Miami; The University of Florida; Rice University; Florida State University; New College; and Stetson University, etc.

There is a cohort of students who have been in the Program's SAT "bubble" for three years now and who are graduating in the Spring of 2024. This group will be submitting college applications in the Fall of 2023. Therefore, many of them will be taking the SAT exam in October and November of 2023. The expectation is that more than a few of them will be accepted into Ivy League Schools, and many more of them will be accepted to top 50 colleges and universities. Additionally, the program has positively impacted many students who have traditionally graduated in the lower 10% of their class. By increasing their SAT

scores, many of these student's college prospects were improved; they were no longer relegated to the singular choice of the local community college.

The principles of this program are also used with The First Star Program at The University of Miami. The First Star organization has cohorts of students in the foster care system that they provide wrap-around services to. One of the services that First Star provides to their students is instruction in SAT mathematics. The majority of the students in this program graduate from highschool and attend college. The mathematics instruction that they receive ensures that they are college ready.

Justification For Building SAT Programs

Districts, schools, and teachers have to intentionally construct sustainable SAT programs that can service their students and families; especially minority and low income students. According to Geiser (2015), race is increasingly becoming a factor that can be used to predict a student's SAT score; this is problematic. According to Buchmann, Condron, and Roscigno (2010), test preparation services tend to give the student who already has an advantage an even greater chance of improving her SAT score. They further state the following: "Social class inequalities in test preparation, particularly costly SAT courses and private tutoring, are notable and have at least moderate consequences for SAT scores and selective college enrollment." Programs that are created inside the public school space will give all students a chance to maximize their SAT potential

According to Julia Isaacs (Brookings Institute, 2012), "Poor children in the United States start school at a disadvantage in terms of their early skills, behaviors, and health. Fewer than half (48 percent) of poor children are ready for school at age five, compared to 75 percent of children from families with moderate and high income, a 27 percentage point gap." SAT Programs can be one of the many solutions to help rectify this great disaprity.

As educators it is imperative that we build SAT systems that are sustainable and effective. The recent Supreme Court ruling that eliminates race as a criteria for college admissions will undoubtedly initiate a decline in certain segments of the population being enrolled at the most prestigious institutions in the United States. Equity and fairness demands that we best equip all of our students to achieve maximal success. School administrators will not design and build these programs (they are busy); they must be built and sustained by dedicated teachers.

Lesson Plan

Context [1]: This lesson plan anticipates heterogeneous cohorts of students. Additionally, it anticipates students with 504 Plans and IEP's.

Standards: MA.912.NSO.1.2 Generate equivalent algebraic expressions using the properties of exponents. **MA.912.AR.3.5** Given the *x*-intercepts and another point on the graph of a quadratic function,write the equation for the function. **MA.912.AR.3.6** Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context. LAFS.910.L.1.1 Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.; LAFS.910.L.1.2 Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Instructional Objectives/Learner Outcomes: The objective is for students to be able to effectively answer SAT questions within the College Board prescribed timeframe. Additionally, this program is specifically designed for students to increase their SAT scores in order to be accepted into the best possible college or university.

Assessment:
Students must be assessed each time that the cohort meets. These assessments must always be College Board approved materials and must be administered according to College Board standards.

Materials/Equipment/Resources: Promethean, laptops, phones, IPADS, pencil, paper, and College Board Approved Calculator

Vocabulary: Linear, Solutions, Zeros, Systems of Equations, Exponential Growth and Decay

Instructional Procedures:

- 15 Minute Lecture: The lecture should be on an SAT focus category. Setting up and solving systems of equations from word problems.
  Investigating how the remainder and factor theorems appear in questions on The SAT.
- 25 minutes: The students should then be given an SAT Math Practice test with College Board approved questions. The answers should be logged in a premade Google Forms.
- The teacher is able to watch student scores and statistics live as the scores are submitted.
- Google Forms effectively disaggregates the data in real time and this information can immediately be used to drive instruction.
- 5 Minutes: The teachers review the data and share some of the information with the students (without revealing student scores). Commonly missed questions are highlighted and discussed with the students prior to working through the solutions.
- 45 minutes: The teacher then uses the Google Forms data as a guide and reviews all

of the questions in granular detail. Student input and perspective must always be encouraged during this time. The questions should be reviewed in descending order: most difficult to least difficult.
- Students should be encouraged to redo questions that were answered incorrectly.
- Students should be reminded of ongoing SAT questions from Khan Academy.

Resource List

- Access to the internet
- Preferably a promethean board (The program still works without it)
- Some application like padlet
- Remind for communication
- Google Forms
- Access to College Board Approved SAT questions
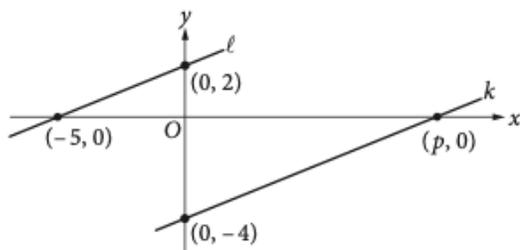- SAT Study Guides (not absolutely necessary
- Khan Academy Account

Practice SAT Questions

5

$$\sqrt{2k^2 + 17} - x = 0$$

If $k > 0$ and $x = 7$ in the equation above, what is the value of k ?

A) 2

B) 3

C) 4

D) 5

6

In the xy-plane above, line ℓ is parallel to line k. What is the value of p ?

A) 4

B) 5

C) 8

D) 10

7

If $\dfrac{x^{a^2}}{x^{b^2}} = x^{16}$, $x > 1$, and $a + b = 2$, what is the value of $a - b$?
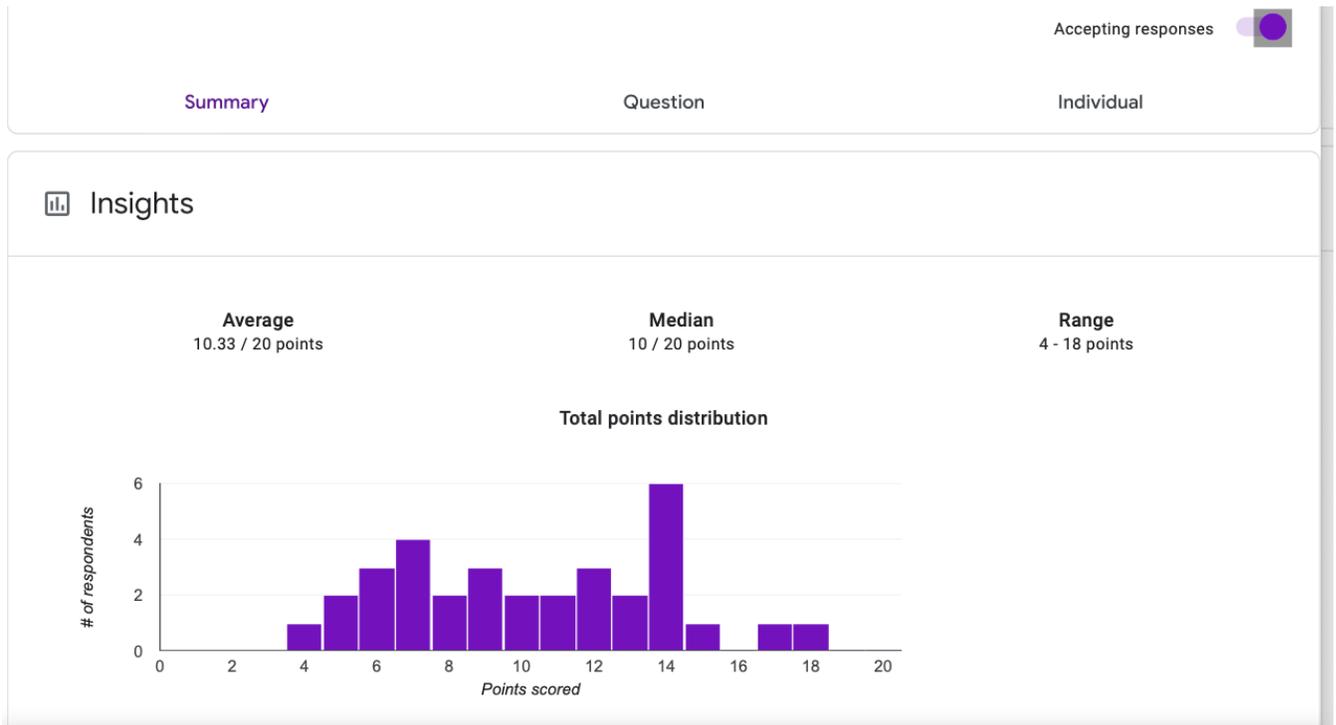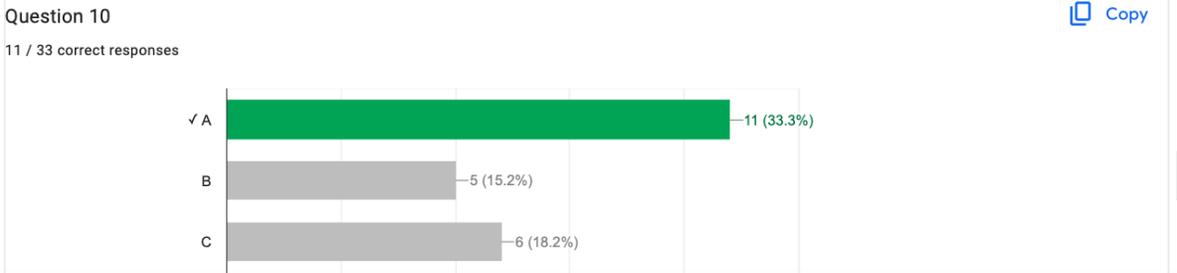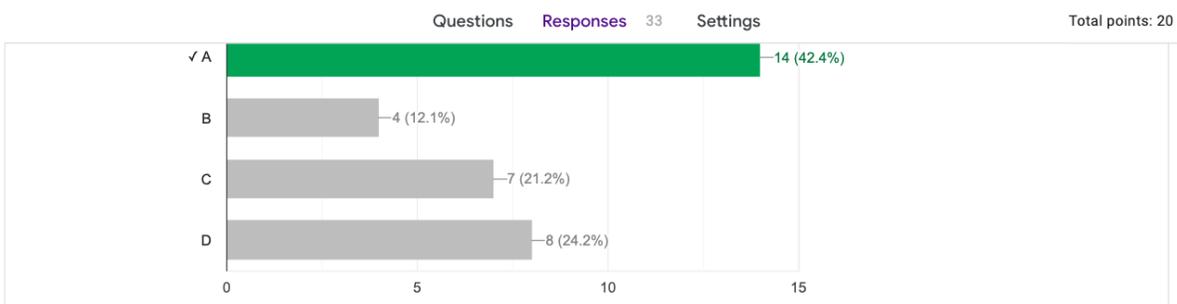
A) 8

B) 14

C) 16

D) 18

8

$$nA = 360$$

The measure A, in degrees, of an exterior angle of a regular polygon is related to the number of sides, n, of the polygon by the formula above. If the measure of an exterior angle of a regular polygon is greater than 50°, what is the greatest number of sides it can have?

A) 5

B) 6

C) 7

D) 8

Google Forms Student Practice Test Data

Summary   Question   Individual

📊 Insights

Average	Median	Range
10.33 / 20 points	10 / 20 points	4 - 18 points

Total points distribution

PT - 1 Section 3

Questions   Responses   33   Settings   Total points: 20

✓ A — 14 (42.4%)
B — 4 (12.1%)
C — 7 (21.2%)
D — 8 (24.2%)

Question 10   Copy

11 / 33 correct responses

✓ A — 11 (33.3%)
B — 5 (15.2%)
C — 6 (18.2%)

Bibliography

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